(IJRSSH) 2019, Vol. No. 9, Issue No. III, Jul-Sep e-ISSN: 2249-4642, p-ISSN: 2454-4671

DETERMINE THE OPTIMAL PRODUCTION PLAN FOR ECONOMIC RESOURCES USING BOUNDED VARIABLES DECISION MAKING

Dr. Haseeb A. Al-shammari
University of Sumer, Iraq

ABSTRACT

Decision making plays an important role in economic, finance, statistics, operations research, engineering, so on. The optimal planning of economic resources requires decision makers in the industrial establishment to use quantitative approaches and mathematical models in the preparation and design of strategies and choose optimal plans. In this paper, a bounded variables quantitative approach is used. It is a more flexible quantitative approach than the unbounded approach, whereby the decision maker can build a mathematical model for determining the optimal production plan for economic resources according to the available resources. This approach helps the decision maker to formulate the mathematical model with decision variables that have upper and lower bounds according to the available resources.

Keywords—Economic resources, ;production plan; lineaer programming, bounded variabels.

1-INTRODUCTION

Economic development requires the advancement of sectors economic and industrial sectors in particular process processors are based on the development of this sector by optimal use of available resources provides the best way to exploit such resources in such a way as to avoid the loss of those resources and provides an opportunity to achieve technical efficiency.

Quantitative approaches paly important role in determining the optimal plan and optimal strategies according to available economic resources. Linear programming approach is the most technique used to find the optimal allocation of economic resources. Aljbory[5] used linear programming model to minimize the cost and maximize the profit General Company for Vegetable Oils. Mathematical programming formulation has presented by Altai [6] to minimize the cost in the General Company for Dairy Products. Abudalsada [7] proposed a linear programming model to achieve the optimal allocation in Glasshouse Farm In Nahrawan. Abdulmajeed [8] presented a mathematical formulation for Production plan in Al-Nu'man General Company. The optimal allocation of economic resources was also studied by [9], [10], [11] using linear programming approach.

On the other hand, the bounded variables approach is a one of the mathematical tools, to solve decision making problems in different areas for taking the efficient, timely and accurate decision. Also, this approach extends the unbounded linear programming formulation to accommodate mathematical programming with upper and lower bounds. Moreover, based on the available economic resources, the decision maker determines the upper and lower bounds for the mathematical formulation of the problem. For more details see [12].

In this paper, we present a bounded variables decision making approach to determine the optimal production plan for economic resources in Algerian Aluminum Company.

2-LINEAR PROGRAMMING APPROACH

Linear programming approach is a one of important mathematical tool for solving decision making problems in different fields such as in economic, finance, statistics and operations research. For more details about linear programming theory and application, we refer to [1][2][3][12][13],where comprehensive details are given. The material of this section can be found in [12].

2.1- Mathematical formulation

The mathematical formulation of linear programming is described as follows:

MaX Z= C′ X
…………(2.1)

AX=b

X >= 0

Where

Z = objective function

A= the matrix of the coefficients associated with decision variables.

x= the decision variable.

b= the associated right hand side values.

2.2- Simplex method

The simplex method is an efficient method to find the optimal solution of any mathematical linear programming model. Now we describe the solution procedure in [12] as follows:

“The Simplex Algorithm (Minimization Problem)

INITIALIZATON STEP

choose a starting basic feasible solution with basis B. (Several procedures for finding an initial basis will be described in Chapter 4.)

MAIN STEP

1- Solve the system $Bx_B = b$ (with unique solution $x_B = B^{-1} b = b^-$).
Let $x_B = b^-$, $x_N = 0$, and $z = c_B x_B$.

2- Solve the system $w_B = c_B$ (with unique solution $w = c_B B^{-1}$). (the vector w is referred to as the vector of simplex multipliers because its components are the multipliers of the rows of A that are added to the objective function in order to bring it into canonical form) Calculate $z_j - c_j = wa_j - c_j$ for all no basic variable. (This is known as the pricing operation.) Let

$$Z_k - C_k = \underset{J \in J}{\text{maximum}} \left\{ z_{j-}\ c_j \right\}$$

Where J is the current set of indices associated with the no basic variables if $z_{j-}\ c_k \leq 0$, then stop with the current basic feasible solution as optimal solution. Otherwise, go to step 3 with x_k as the entering variable . (the strategy for selecting an entering variable is known as Danzig's rule).

3- Solve the system $By_k = a_k$ (with unique solution $y_k = B^{-1} a_k$) . if $y_k \leq 0$, then stop with the conclusion the optimal solution is unbounded along the ray

$$\left\{ \begin{bmatrix} b^- \\ 0 \end{bmatrix} + x_k \begin{bmatrix} -y_k \\ e_k \end{bmatrix} : \ x_k \ \geq 0 \right\}$$

Where e_k is an (n- m) – vector of zeros except for a 1 at the kth position . if $y_k \leq 0$, go to step 4.

4- Let x_k enter the basis . The index r of the blocking variable , xB_R which leaves the basis is determined by the following minimum ratio Test:

$$\frac{b_r^-}{y_{rk}} = \text{minimum} \left\{ \frac{b_i^-}{y_{ik}} : \ y_{ik} \ > 0 \right\}.$$

Update the basis B where a_k replaces a_{B_r} , update the index set J , and repeat Step 1”.

3- BOUNDED VARIABLES LINEAR PROGRAMMING APPROACH

Many real life applications require the decision making variables to have upper bounds and lower bounds. which helps to reach more accurate optimal plan. This kind of optimization problems is called bounded variables decision making problems. For more details we refer to [12].

3.1- Mathematical formulation

The bounded variables linear programming model is described as follows:

MaX Z= C′ X
…………(3.1)

(IJRSSH) 2019, Vol. No. 9, Issue No. III, Jul-Sep e-ISSN: 2249-4642, p-ISSN: 2454-4671

AX=b

l≤X ≤u

Where

Z = objective function

A= the matrix of the coefficients associated with decision variables.

x= the decision variable.

b= the associated right hand side values.

l: the lower bound of the decision variable.

u: the upper bound of the decision variable.

3.2- Bounded variables simplex method

The bounded variables simplex method is given in [12] as follows:

INITIALIZATION STEP"

Find a starting basis feasible solution (use artificial variables if necessary).Let x_B

be the basic variables and let x_{N_1} and x_{N_2} be the nonbasic variables at their lower

and upper bounds, respectively, Form the following tableau where $\hat{z}$ = $c_B B^{-1}$b +

(c_{N_1}-$c_B B^{-1}N_1)l_{N_1}$ + (c_{N_1}-$c_B B^{-1}N_2)u_{N_2}$ and $\hat{b}$ = B^{-1}b - $B^{-1}N_1 l_{N_1}$ - $B^{-1}N_2 u_{N_2}$):

Special Simplex Implementations and optimality conditions

	z	x_B	x_{N_1} x_{N_2}	RHS
z	1	0	$c_B B^{-1}N_1$-c_{N_1} $c_B B^{-1}N_2$ -c_{N_2}	$\hat{z}$
x_B	0	I	$B^{-1}N_1 B^{-1}N_2$	$\hat{b}$

MAIN STEP

1- If z_j - c_j ≤ 0 for nonbasic variables at their lower bounds and z_j - c_j≥ 0 for non-basic variables at their upper bounds, then the current solution is optimal. Otherwise, if one of these conditions is violated for the index k, then go to step 2 if x_k is at is lower bound and Step 3 if x_k is at its upper bound.

2- The variable x_k is increased from its current value of l_k to l_k + Δ_k. The value of Δ_k is given by equation (1) where $\boldsymbol{\alpha_1}$ and $\boldsymbol{\alpha_2}$ are given by equations (2) and (3) as follows:

Δ_k = minimum { α_1, α_2, $u_k - l_k$} …………..(1)

$$\boldsymbol{\alpha_1} = \begin{cases} \underset{1\le i\le m}{minimum}\left\{\frac{\hat{b_i}-l_{B_i}}{y_{ik}} : y_{ik} > 0\right\} = \frac{\hat{b}_r - l_{B_r}}{y_{rk}} & iF\ y_k \le 0 \\ \infty & iF\ y_k \le 0 \end{cases} \quad (2)$$

$$\boldsymbol{\alpha_2} = \begin{cases} \underset{1\le i\le m}{minimum}\left\{\frac{u_{B_i}-b_i}{-y_{ik}} : y_{ik} < 0\right\} = \frac{u_{B_i}-b_i}{-y_{ik}} & iF\ y_k \ge 0 \\ \infty & iF\ y_k \ge 0 \end{cases} \quad \ldots(3)$$

if Δ_k=∞, stop ; the optimal objective value is unbounded. Otherwise, the tableau is updated as described previously. Repeat step 1.

3- The variable x_k is decreased from its current value of u_k to u_k - Δ_k. The value of Δ_k is given by equation (4) where $\boldsymbol{\alpha_1}$ and $\boldsymbol{\alpha_2}$ are given by equations (5) and (6) as follows:

Δ_k = minimum { α_1, α_2, $u_k - l_k$} …………..(4)

$$\boldsymbol{\alpha_1} = \begin{cases} \underset{1\le i\le m}{minimum}\left\{\frac{\hat{b_i}-l_{B_i}}{-y_{ik}} : y_{ik} < 0\right\} = \frac{\hat{b}_r - l_{B_r}}{-y_{rk}} & iF\ y_k \ge 0 \\ \infty & iF\ y_k \ge 0 \end{cases} \quad \ldots(5)$$

$$\boldsymbol{\alpha_2} = \begin{cases} minimum \left\{\frac{u_{B_i} - b_i}{y_{ik}} : y_{ik} > 0\right\} = \frac{u_{B_i} - b_i}{y_{ik}} & iF\ y_k \le 0 \\ 1 \le i \le m & \\ \infty & iF\ y_k \le 0 \end{cases}. (6)$$

if Δ_k=∞, stop ; the optimal objective value is unbounded. Otherwise, the tableau is updated as described previously. Repeat step 1".

4-CASE STUDY

The proposed bounded variables approach is applied in Algerian Aluminum Company to determine the optimal production plan of economic resources. We extend the linear programming formulation that is presented by Rabeh [14] to a new mathematical formulation using bounded variables approach based on the objectives of the company and according to the available economic resources. This company aims to achieve the optimal production plan that leads to a maximum profit according to available economic resources.

The mathematical formulation of this problem is described as follows:

Max $Z_P = 304.18X_1 + 281.65X_2 + 291.15X_3 + 348.58X_4$

ST:

$0.0012X_1 + 0.0012X_2 + 0.0012X_3 + 0.0012X_4 \le 3680$

$0.0022X_1 + 0.0022X_2 + 0.0022X_3 + 0.0022X_4 \le 7360$

$0X_1 + 0.0025X_2 + 0X_3 + 0.0025X_4 \le 3680$

$0X_1 + 0X_2 + 0.0060X_3 + 0X_4 \le 7360$

$X_1 + X_2 + X_3 + X_4 \le 3705507$

$0.037X_4 \le 2200$

$0.2198X_2 + 0.2198X_4 \le 313762$

$0.1481X_2 + 0.1481X_4 \le 211442$

$0.0159X_2 + 0.0159X_4 \le 22732$

$0.0105X_2 + 0.0105X_4 \le 15000$

$0.0157X_3 \le 29340$

$0.0048X_3 \le 6194$

$0.0225X_3 \le 23275$

$0.0062X_3 \le 12458$

$0.0040X_3 \le 3660$

$100 \le X_1 \le 547100,\ 200 \le X_2 \le 1567312,\ 300 \le X_3 \le 682430,\ 50 \le X_4 \le 55500$

Now, we solved this model using WINQSB, and we get the following:

Table (1) shows the mathematical model

h2

Maximize	304.18X1 + 281.65X2 + 291.15X3 + 348.58X4
	OBJ/Constraint/Bound
Maximize	304.18X1 + 281.65X2 + 291.15X3 + 348.58X4
C1	0.0012X1 + 0.0012X2 + 0.0012X3 + 0.0012X4 <=3680
C2	0.0022X1 +0.0022 X2 +0.0022 X3 +0.0022 X4 <= 7360
C3	0X1 + 0.0025X2 + 0X3 + 0.0025X4 <= 3680
C4	0X1 + 0X2 + 0.0060X3 + 0X4 <= 7360
C5	X1 + X2 + X3 + X4 <= 3705507
C6	0.037 X4 <= 2200
C7	0.2198 X2 + 0.2198 X4 <= 313762
C8	0.1481 X2 + 0.1481X4 <= 211442
C9	0.0159 X2 + 0.0159 X4 <= 22732
C10	0.0105 X2 + 0.0105X4 <= 15000
C11	0.0157 X3 <= 29340
C12	0.0048 X3 <= 6194
C13	0.0225X3 <= 23275
C14	0.0062X3 <= 12458
C15	0.0040X3 <= 3660
Integer:	
Binary:	
Unrestricted:	
X1	>=100, <=547100
X2	>=200, <=1371919
X3	>=300, <=682430
X4	>=50, <=55570

Table (2) shows the results

	13:49:43		Friday	June	14	2019		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	X1	547,100.0000	304.1800	166,416,900.0000	0	basic	0	M
2	X2	1,371,919.0000	281.6500	386,400,900.0000	0	basic	0	348.5800
3	X3	682,430.0000	291.1500	198,689,500.0000	0	basic	0	M
4	X4	55,570.0000	348.5800	19,370,590.0000	0	basic	281.6500	M
	Objective	Function	(Max.) =	770,877,800.0000				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	3,188.4230	<=	3,680.0000	491.5773	0	3,188.4230	M
2	C2	5,845.4410	<=	7,360.0000	1,514.5590	0	5,845.4410	M
3	C3	3,568.7210	<=	3,680.0000	111.2787	0	3,568.7210	M
4	C4	4,094.5800	<=	7,360.0000	3,265.4200	0	4,094.5800	M
5	C5	2,657,019.0000	<=	3,705,507.0000	1,048,488.0000	0	2,657,019.0000	M
6	C6	2,056.0900	<=	2,200.0000	143.9099	0	2,056.0900	M
7	C7	313,762.0000	<=	313,762.0000	0	1,281.3920	12,258.2500	313,762.1000
8	C8	211,411.1000	<=	211,442.0000	30.9313	0	211,411.1000	M
9	C9	22,697.0700	<=	22,732.0000	34.9308	0	22,697.0700	M
10	C10	14,988.6300	<=	15,000.0000	11.3702	0	14,988.6300	M
11	C11	10,714.1500	<=	29,340.0000	18,625.8500	0	10,714.1500	M
12	C12	3,275.6640	<=	6,194.0000	2,918.3360	0	3,275.6640	M
13	C13	15,354.6800	<=	23,275.0000	7,920.3240	0	15,354.6800	M
14	C14	4,231.0660	<=	12,458.0000	8,226.9340	0	4,231.0660	M
15	C15	2,729.7200	<=	3,660.0000	930.2799	0	2,729.7200	M

5-CONCLUSION

A bounded variables qualitative approach has been presented to determine the optimal production plan of economic resources. Also, the proposed approach is based on the upper and lower bounds of the decision variables, using this property the decision maker can set up the optimal production paln according to available economic resources. Moreover, the company can determine the optimal production plan so that this will lead to the maximum profit according to available economic resources.

REFERENCES

[1] Al-Salih, R. and Bohner, M. Separated and state-constrained separated linear programming problems on time scales. Bol. Soc.Parana. Mat. (3), vol. 38, 4: 181-195, 2020.

[2] Al-Salih, R. and Bohner, M. Linear programming problems on time scales. Appl. Anal. Discrete Math., 12:192-204, 2018.

[3] Al Salih, R., Habeeb, A. and Laith, W. 2019. A Quantum Calculus Analogue of Dynamic Leontief Production Model with Quadratic Objective Function. Journal of Engineering and Applied Sciences, 14: 6415-6418.

[5] Aljbory, K. H. Select the optimal commodity using linear programming, MS. Thesis, Almustansiriya University 2001.

[6]Altai, Sh. Optimal allocation for economic resources using linear programming. MS. Thesis, Baghdad University, 2001.

[7] Abudalsada, B. Financial Economic Assessment of the Glasshouse Farm In Nahrawan, to determine the optimal use of farm resources. PhD Thesis, Almustansiriya University, 2005.

[8] Abdulmajeed, S. Industrial Production Planning Using Linear programming approach, an applied study in Al-Nu'man General Company. MS. Thesis, Almustansiriya University, 2004.

[9]Almisraky, J. Preparing and evaluating the production plan at the Diwaniya Textile Factory Using quantitative input. MS. Thesis, Al-Qadisiyah University, 2001.

[10]Alsakini, A. Allocation of economic resources by using linear programming method. MS. Thesis, Baghdad University, 2008.

[11]Albermany, S. Using the linear programming method to determine the mixed Optimal Production of Wasit General Textile Industries. Journal of Administration and Economics, 2011,No. 63, pp. 130-166.

[12] Bazaraa M. S. , Jarvis J. J. and (2010) Linear Programming and Network Flows, A John Wiley & Sons, Inc., Publication.

[13] Charnes, A.; Cooper, W. Management Models and Industrial Applications of Linear Programming; John Wiley and Sons: New York, NY, USA, 1961.

[14]Rabeh, B. Linear programming and its role in preparing the optimal plan in the economic institution,: case study in ALGAL, journal of economic science, 5: 112-131,2005.